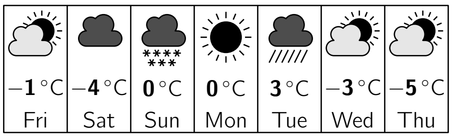
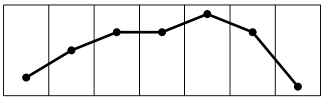
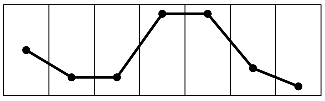
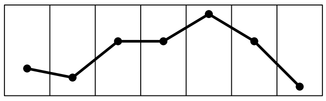
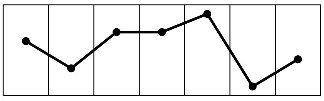
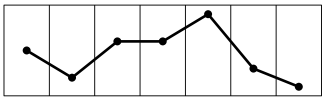
3 points

 # 1. Paula's weather app shows a diagram of the predicted weather and maximum temperatures for the next seven days, as shown. Which of the following represents the corresponding graph of maximum temperatures?



(**A**)  (**B**)  (**C**) 

(**D**)  (**E**) 

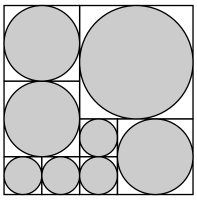
 # 2. How many integers are in the interval $(20 - \sqrt{21}, 20 + \sqrt{21})$?

(**A**) 9 (**B**) 10 (**C**) 11 (**D**) 12 (**E**) 13

 # 3. A cube with edge 1 is cut into two identical cuboids. What is the surface area of one of these cuboids?

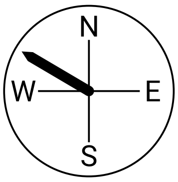
(**A**) $\frac{3}{2}$ (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

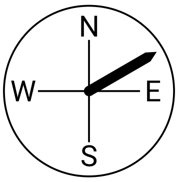
 # 4. A large square is divided into smaller squares, as shown. A shaded circle is inscribed inside each of the smaller squares. What proportion of the area of the large square is shaded?

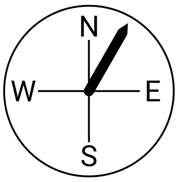


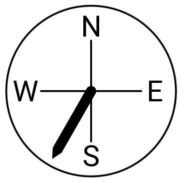
(**A**) $\frac{8\pi}{9}$ (**B**) $\frac{13\pi}{16}$ (**C**) $\frac{3}{\pi}$ (**D**) $\frac{3}{4}$ (**E**) $\frac{\pi}{4}$

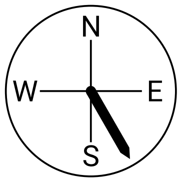
 # 5. After the storm last night, the flagpole on our school building is leaning over. Looking from northwest, its tip is to the right of its bottom point. Looking from the east, its tip is also to the right of its bottom point. In which direction could the flagpole be leaning over?

(**A**) 

(**B**) 

(**C**) 

(**D**) 

(**E**) 

 # 6. A rectangular sheet of paper has length x and width y, where $x > y$. The rectangle may be folded to form the curved surface of a circular cylinder in two different ways. What is the ratio of the volume of the longer cylinder to the volume of the shorter cylinder?

(**A**) $y^2 : x^2$ (**B**) $y : x$ (**C**) 1:1 (**D**) $x : y$ (**E**) $x^2 : y^2$

 # 7. Let $x = \frac{\pi}{4}$. Which of the following numbers is the largest?

(**A**) x^4 (**B**) x^2 (**C**) x (**D**) $\sqrt{x}$ (**E**) $\sqrt[4]{x}$

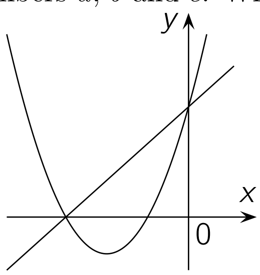
 # 8. How many 3-digit-numbers formed using only the digits 1, 3 and 5 are divisible by 3? You may use digits more than once.

(**A**) 3 (**B**) 6 (**C**) 9 (**D**) 18 (**E**) 27

 # 9. What is the area of the triangle whose vertices are at (p, q), $(3p, q)$ and $(2p, 3q)$, where $p, q > 0$?

(**A**) $\frac{pq}{2}$ (**B**) pq (**C**) $2pq$ (**D**) $3pq$ (**E**) $4pq$

 # 10. The parabola in the figure has an equation of the form $y = ax^2 + bx + c$ for some distinct real numbers a, b and c. Which of the following equations could be an equation of the line in the figure?



(**A**) $y = bx + c$ (**B**) $y = cx + b$ (**C**) $y = ax + b$ (**D**) $y = ax + c$ (**E**) $y = cx + a$

4 points

 # 11. What proportion of all the divisors of 7! is odd?

(**A**) $\frac{1}{2}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{1}{5}$ (**E**) $\frac{1}{6}$

 # 12. If $A = (0,\ 1) \cup (2,\ 3)$ and $B = (1,\ 2) \cup (3,\ 4)$, what is the set of all numbers of the form $a + b$ with a in A and b in B?

(**A**) $(1,\ 7)$ (**B**) $(1,\ 5) \cup (5,\ 7)$ (**C**) $(1,\ 3) \cup (3,\ 7)$
(**D**) $(1,\ 3) \cup (3,\ 5) \cup (5,\ 7)$ (**E**) none of the previous

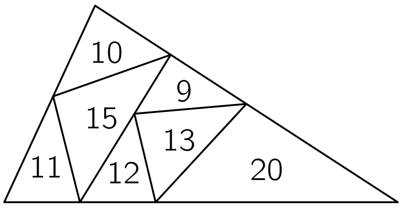
 # 13. How many three-digit natural numbers have the property that when their digits are written in reverse order, the result is a three-digit number which is 99 more than the original number?

(**A**) 8 (**B**) 64 (**C**) 72 (**D**) 80 (**E**) 81

 # 14. The first 1000 positive integers are written in a row in some order and all sums of any three adjacent numbers are calculated. What is the greatest number of odd sums that can be obtained?

(**A**) 997 (**B**) 996 (**C**) 995 (**D**) 994 (**E**) 993

 # 15. A large triangle is divided into smaller triangles as shown. The number inside each small triangle indicates its perimeter. What is the perimeter of the large triangle?



(A) 31 **(B)** 34 **(C)** 41 **(D)** 62

(E) none of the previous

 # 16. For a positive integer N, we denote by $p(N)$ the product of the digits of N when written in decimal form. For example, $p(23) = 2 \times 3 = 6$. What is the value of the sum $p(10) + p(11) + p(12) + ... + p(99) + p(100)$?

(A) 2025 **(B)** 4500 **(C)** 5005 **(D)** 5050

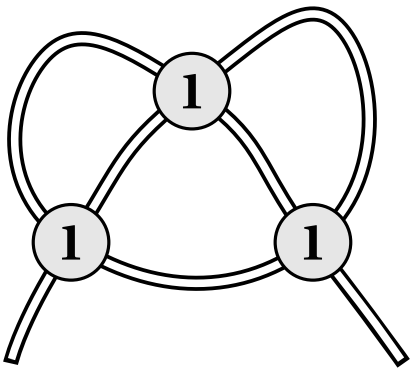
(E) none of the previous

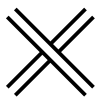
 # 17. In the 5×5 square shown the sum of the numbers in each row and in each column is the same. There is a number in every cell, but some of the numbers are not shown. What is the number in the cell marked with a question mark?

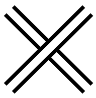
	16		22	
20		21		2
	25		1	
24		5		6
	4		?	

(A) 8 **(B)** 10 **(C)** 12 **(D)** 18 **(E)** 23

 # 18. A piece of string is lying on the table. It is partially covered by three coins as seen in the figure.



Under each coin the string is equally likely to pass over itself like this: 

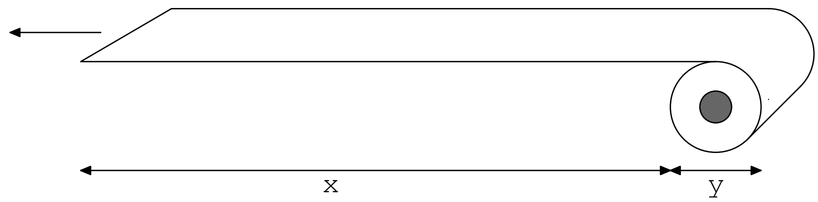
or like this: .

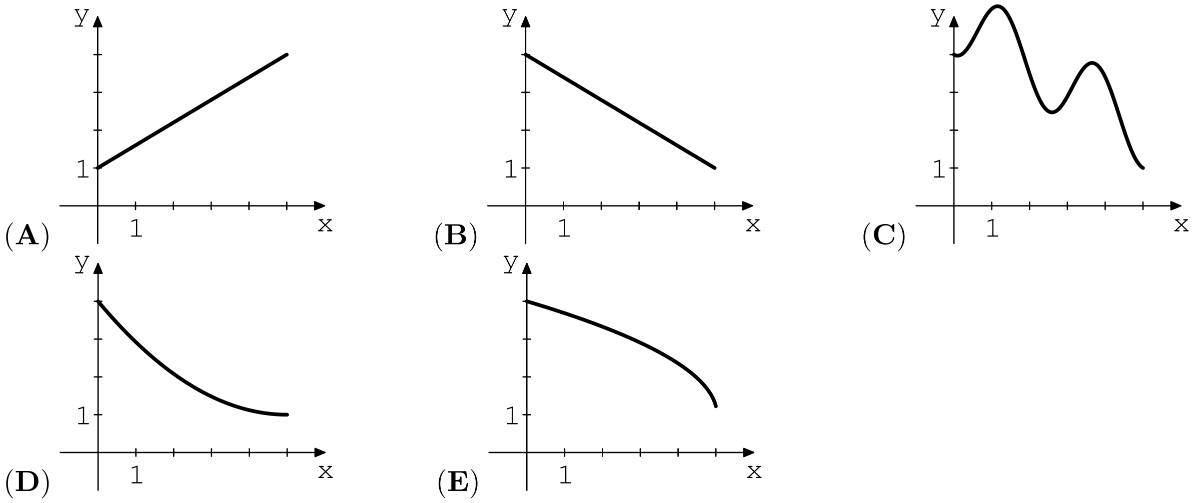
What is the probability that the string is knotted after its ends are pulled?

(A) $\frac{1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{3}{8}$

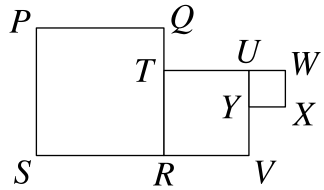
 # 19. A naughty pup grabs the end of a roll of toilet paper and walks away at a constant speed.

Which of the functions below best describes the thickness y of the roll as a function of the unrolled part x?





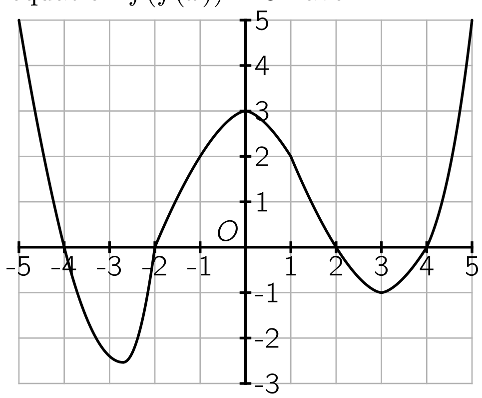
 # 20. The diagram shows three squares, $PQRS$, $TRVU$ and $UWXY$. They are placed together, edge to edge. Points P, T and X lie on the same same straight line. The area of $PQRS$ is 36 and the area of $TRVU$ is 16. What is the area of triangle PXV?



(**A**) $14\frac{2}{3}$ (**B**) $15\frac{1}{3}$ (**C**) 16 (**D**) $17\frac{2}{3}$ (**E**) 18

5 points

 # 21. The figure shows the graph of a function $f : [-5, 5] \to \mathbb{R}$. How many distinct solutions does the equation $f(f(x)) = 0$ have?



(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 7 (**E**) 8

 # 22. The numbers 1, 2, 7, 9, 10, 15 and 19 are written down on a blackboard. Two players alternately delete one number each until only one number remains on the blackboard. The sum of the numbers deleted by one of the players is twice the sum of the numbers deleted by the other player. What is the number that remains?

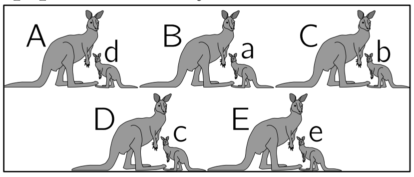
(**A**) 7 (**B**) 9 (**C**) 10 (**D**) 15 (**E**) 19

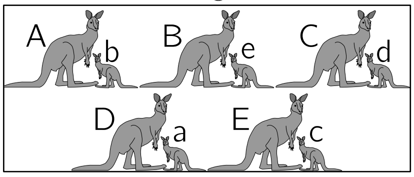
 # 23. The function $f(x)$ is such that $f(x+y) = f(x) \cdot f(y)$ and $f(1) = 2$. What is the value of $\dfrac{f(2)}{f(1)} + \dfrac{f(3)}{f(2)} + \cdots + \dfrac{f(2021)}{f(2020)}$?

(**A**) 0 (**B**) $\dfrac{1}{2}$ (**C**) 2 (**D**) 2020

(**E**) none of the previous

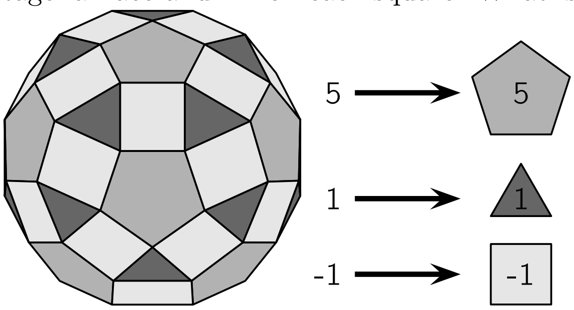
 # 24. Five kangaroos named A, B, C, D and E have one child each, named a, b, c, d and e. In the first group photo shown exactly two of the children are standing next to their mothers. In the second group photo exactly three of the children are standing next to their mothers. Whose child is a?





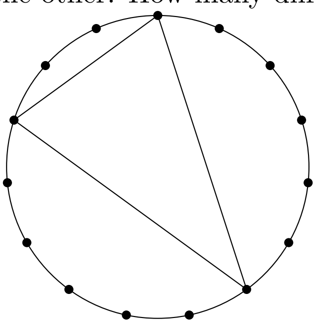
(**A**) A (**B**) B (**C**) C (**D**) D (**E**) E

 # 25. The solid shown in the diagram has 12 regular pentagonal faces, the other faces being either equilateral triangles or squares. Each pentagonal face is surrounded by 5 square faces and each triangular face is surrounded by 3 square faces. John writes 1 on each triangular face, 5 on each pentagonal face and −1 on each square. What is the total of the numbers written on the solid?



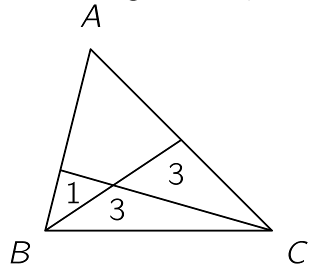
(**A**) 20 (**B**) 50 (**C**) 60 (**D**) 80 (**E**) 120

 # 26. On a circle 15 points are equally spaced. We can form triangles by joining any three of these. We count two triangles as being the same if they are congruent i.e. one is a rotation and/or a reflection of the other. How many different triangles can be drawn?



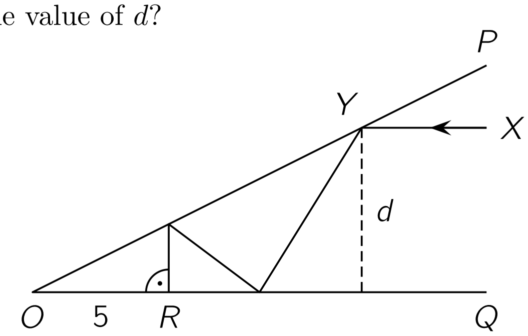
(**A**) 19 (**B**) 91 (**C**) 46 (**D**) 455 (**E**) 23

 # 27. A triangle ABC is divided into four parts by two straight lines, as shown. The areas of the smaller triangles are 1, 3 and 3. What is the area of the original triangle?



(**A**) 12 (**B**) 12.5 (**C**) 13 (**D**) 13.5 (**E**) 14

 # 28. Two plane mirrors OP and OQ are inclined at an acute angle (diagram is not to scale). A ray of light XY parallel to QO strikes mirror OP at Y. The ray is reflected and hits mirror OQ, is reflected again and hits mirror OP and is reflected for a third time and strikes mirror OQ at right angles at R, as shown. The distance OR is 5 cm. The ray XY is d cm from the mirror OQ. What is the value of d?



(**A**) 4 (**B**) 4.5 (**C**) 5 (**D**) 5.5 (**E**) 6

 # 29. Let $M(k)$ be the maximum value of $|4x^2 - 4x + k|$ for x in the interval $[-1, 1]$, where k can be any real number. What is the minimum possible value of $M(k)$?

(**A**) 4 (**B**) 9/2 (**C**) 5 (**D**) 11/2 (**E**) 8

 # 30. A certain game is won when one player gets 3 points ahead. Two players A and B are playing the game and at a particular point, A is 1 point ahead. Each player has an equal probability of winning each point. What is the probability that A wins the game?

(**A**) $\frac{1}{2}$ (**B**) $\frac{2}{3}$ (**C**) $\frac{3}{4}$ (**D**) $\frac{4}{5}$ (**E**) $\frac{5}{6}$